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Research Article

STUDY OF AXIALLY SYMMETRIC ANISOTROPIC MASSIVE STRING COSMOLOGICAL MODEL IN F(R, T) THEORY OF GRAVITY

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ABSTRACT

Axially symmetric anisotropic cosmological model with massive string fluid source are obtained in f(R,T) theory of Gravity proposed by Harko *et al*.(Phys.Rev.D 84,02420,2011).The model which we obtained study geometric (Nambu) string and Reddy string in f(R,T) theory of gravity. Some physical properties of the model are also discussed.

INTRODUCTION

Modified theories of gravitation have been extensively studied due to their cosmological implications. Noteworthy among them are the scalar-tensor theories of gravitation formulated by Brans and Dicke(1961),Sen (1957),Sen and Dunn(1971) and Saez and Ballester(1986).In Recent years there has been an increasing interest in modified theories of gravity in view of the direct evidence of late time acceleration of the universe and existence of the dark matter and dark energy(Reiss *et al*.1998;Perlmutter *et al*.1999;Bennet *et al*.2003).In particular F(R) theory of gravity formulated by Nojiri and Odintsov (2003a) and F(R,T) theory of gravity proposed by Harko *et al*.(2011) are attracting more and more attention. It has been suggested that cosmic acceleration can be achieved by replacing Einstein –Hilbert action of general relativity with a general function f(R)(R being the Ricci scalar curvature). A comprehensive review of modified f(R) gravity is given by Copeland *et al*.(2006) while a detailed discussion of f(R,T) gravity is given by Harko *et al*.(2011).Also, Caroll *et al*.(2004), Nojiri and Odintsov (2003b,2004,2007) and Chiba *et al*.(2007) are some of the authors who have investigated several aspects of f(R) gravity.

Moreover in recent years there has been a lot of interest in cosmic strings and string cosmological models. Cosmic strings are one dimensional topological defect associated with spontaneous breaking of symmetry of the universe whose possible production site is cosmological phase transition in the early state of the universe(Kibble(1976) and Vilenkin (1985)).Letelier(1983),Vilenkin(1981),Krori *et al*.(1990,1994) are some authors who initiated the study of string cosmological models. Adhav (2012), Reddy *et al*. (2012a) Sahoo P.K (2014) Choubey and Shukla (2013) Ghate H.R. and Sontakke A.S (2014) presented some cosmological models in f(R, T) theory of gravity.

Motivated by the above investigations we study, in this paper, axially symmetric anisotropic massive string cosmological model in f(R,T) theory of gravity. Cosmological model we obtained represent Nambu string and Reddy string.

METRIC AND FIELD EQUATIONS

We consider uniform, anisotropic and axially symmetric metric (Bhattacharya and Karade 1993)in the form

$$ds^2 = dt^2 - A^2(t)[d\chi^2 + f^2(\chi)d\Phi^2] - B^2(t)dz^2 \tag{2.1}$$

With the convention $x^1 = \chi, x^2 = \Phi, x^3 = z$ and $x^4 = t$.Also A and B are function of cosmic time t alone while f is a function of the co-ordinate χ alone.

Field equations in f(R, T) theory of gravity for the function f(R, T) =R+2f (T) (Harko *et al*. (2011)) are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{2.2}$$

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Where f(T) is an arbitrary function of trace of the stress energy tensor of matter and overhead prime indicates differentiation with respect to argument.
The energy momentum tensor for a cloud of massive string and perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} - \lambda x^i x_j \tag{2.3}$$

where ρ is the proper energy density of the cloud of strings with massive particle attached to them.

ρ= ρ_p+λ (2.4)

where ρ_p rest energy of particle attached to string .λ is the string tension density of the system of the string. μ^i is the four velocity for the cloud particle,x^i is the four vector which represents the string direction which is the direction of anisotropy also. Moreover the direction of string satisfy the standard relations

$$u^i u_i = -x^i x_i = 1, u^i x_i = 0 x^i = (0,0,B^{-1},0) \tag{2.5}$$

Using co-moving co-ordinate and a particular choice of function given by

F (T) = μT, (2.6)

where μ is constant,
The field equations (2.2) for the metric (2.1) with the help of equations (2.3) and (2.6) yield following independent equations

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} = (8\pi + 3\mu)p + (\lambda - \rho)\mu \tag{2.7}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{1}{A^2}\left(\frac{f''}{f}\right) = (8\pi + 3\mu)(p + \lambda) - \mu\rho \tag{2.8}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2}\left(\frac{f''}{f}\right) = -(8\pi + 3\mu)\rho + (p + \lambda)\mu \tag{2.9}$$

Now, functional dependence of the metric imply

$$\frac{f''}{f} = k^2 \ and\ k^2 = constant \tag{2.10}$$

If k = 0 then $f(\chi) = k_1\chi + k_2$,0 <χ< ∞. Here k1and k2 are constant of integration. Without loss of generality we can take k1 = 1and k2=0.Then we will get f (χ) = χresulting in flat model of the universe.

Now field equations (2.7)-(2.9) reduces to

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} = (8\pi + 3\mu)p + (\lambda - \rho \tag{2.11}$$

$$\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = (8\pi + 3\mu)(p + \lambda) - \mu\rho \tag{2.12}$$

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = -(8\pi + 3\mu)\rho + (p + \lambda)\mu \tag{2.13}$$

where over dot represent differentiation w.r.t. to t.

Solution of the field equations

Field equations we obtained is system of three independent equation with five unknown A, B, p, λ and ρ.Therefore to solve them, we need two more physical assumption amongst the unknown parameter.

Firstly we assume that expansion (θ) is proportional to the components of shear tensor (σ) which also represent anisotropy of the universe. This leads to a polynomial relation between the metric coefficients

$$A = B^m \tag{3.1}$$

where m≠0 is proportionality constant.

We take help of special law of variation proposed by Bermann (1983) for Hubble parameter that yield constant value of DP.
We consider constant deceleration parameter model defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = constant, \tag{3.2}$$

where scale factor R is given by

$$R = (A^2B)^{\frac{1}{3}}. \tag{3.3}$$

Solving equation (3.2) we get

$$R = (at + b)^{\frac{1}{1+q}} \tag{3.4}$$

Where a (≠0) and b are constant of integration and equation implies condition for expansion of the universe i.e. 1+q > 0

Further we need one more relation to solve the system of highly nonlinear equations having more number of unknown than number of equations. Hence we assume relation between physical parameter in term of equation of state for string.

Case (i): Geometric string

Here we consider the equation of state of geometric (Nambu) string as

ρ = λ (3.5)

Then with the help of(3.3) and (3.4)field equations (2.11)–(2.13) admit an exact solution

$$A = (at + b)^{\frac{3}{\alpha\beta}} \tag{3.7}$$

$$B = (at + b)^{\frac{3(\beta-1)}{2\alpha\beta}} \tag{3.8}$$

where $\alpha = (q + 1) and\ \beta = (2m + 1)$are constants and a and b are integrating constants. So without loss of generality we take a=1and b=0then corresponding string model of the solution through proper choice of constants and co-ordinate can be written as

$$ds^2 = dt^2 - t^{\frac{6}{\alpha\beta}}[d\chi^2 + f^2(\chi)d\phi^2] - t^{\frac{3(\beta-1)}{\alpha\beta}}dz^2 \tag{3.9}$$

Equation (3.9) represent axially symmetric anisotropic geometric (Nambu) string cosmological model in framework of f(R, T) theory of gravity.

Physical properties with discussion:
Special volume V is,

$$V = (A^2B) = t^{\frac{3}{\alpha}} \tag{4.1}$$

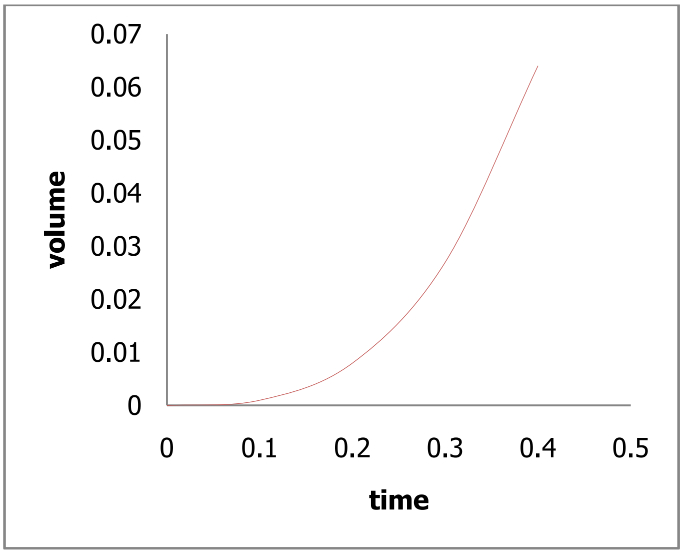

Fig 1: The plot of volume (v) verses time(t), α=1, β=5.

From the figure, it is observed that at an initial epoch, spacial volume is zero and increases with increase in cosmic time t, which shows that universe start evolving with zero volume and expands with cosmic time t showing late time accelerated expansion of the universe.

Expansion scalar,

$$\theta = \frac{3}{\alpha t} \qquad (4.2)$$

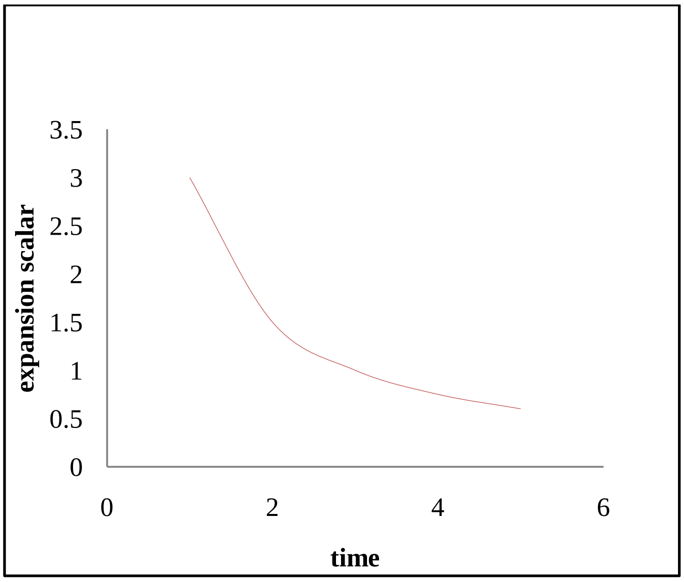

Fig 2: The plot of Expansion scalar (θ) verses time (t), α=1, β=5.

From the figure,it is observed that Expansion scalar at an initial epoch diverges. As cosmic time t increases gradually expansion scalar decreases and finally vanishes when t –› ∞, which shows that it possess late time singularity. Further model has non zero expansion rate i.e. universe start with an infinite rate of expansion .This behaves like big bang model of the universe

Shear scalar,

$$\sigma^2 = \frac{3}{\alpha^2 t^2} \qquad (4.3)$$

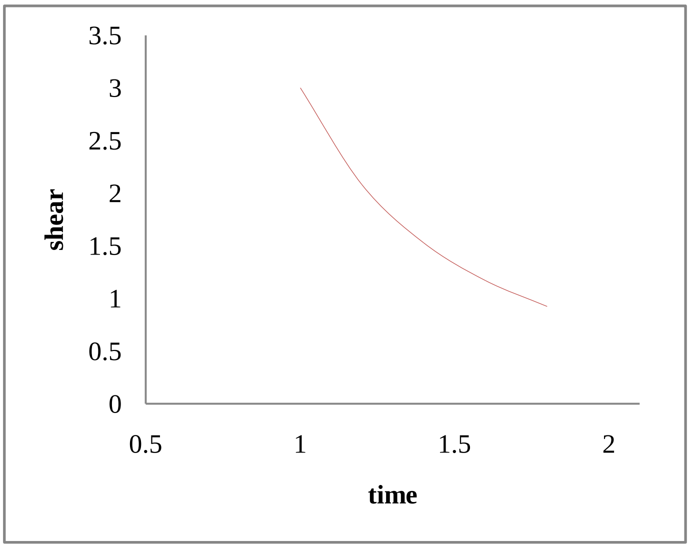

Fig 3: The plot of Shear scalar (σ) verses time (t), α=1, β=5.

From the figure,it is observed that Shear scalar has initial singularity at t = 0 and it dies out for large value of t.

Isotropic pressure,

$$p = \frac{3[3\beta^2+2\alpha\beta(\beta-1)+9]}{4\alpha^2\beta^2(8\pi+3\mu)t^2} \qquad (4.4)$$

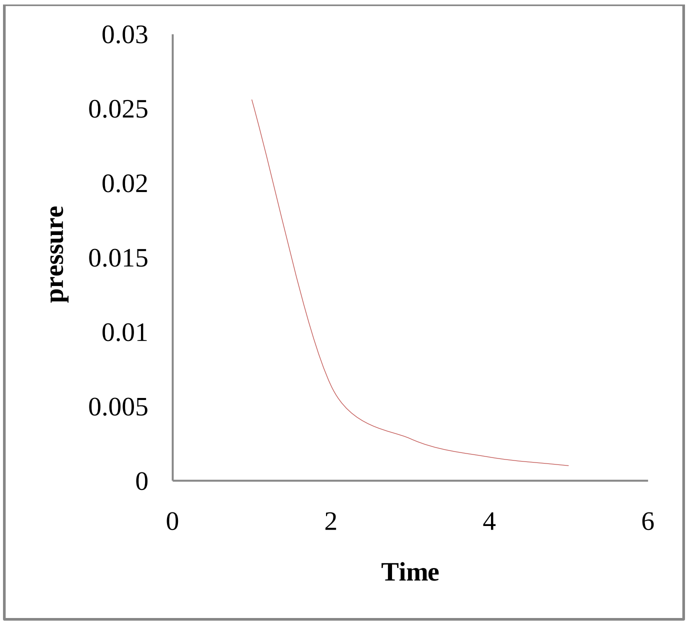

Fig 4: The plot of pressure (p) verses time (t), α=1, β=5.

From the figure, it is observed that Isotropic pressure has initial singularity at t = 0.and it is get vanished as cosmic time t–› ∞.

Proper energy density,

$$\rho = \frac{9[(\alpha-3)-\beta(\alpha-1)]}{4\alpha^2\beta(4\pi+\mu)t^2} \qquad (4.5)$$

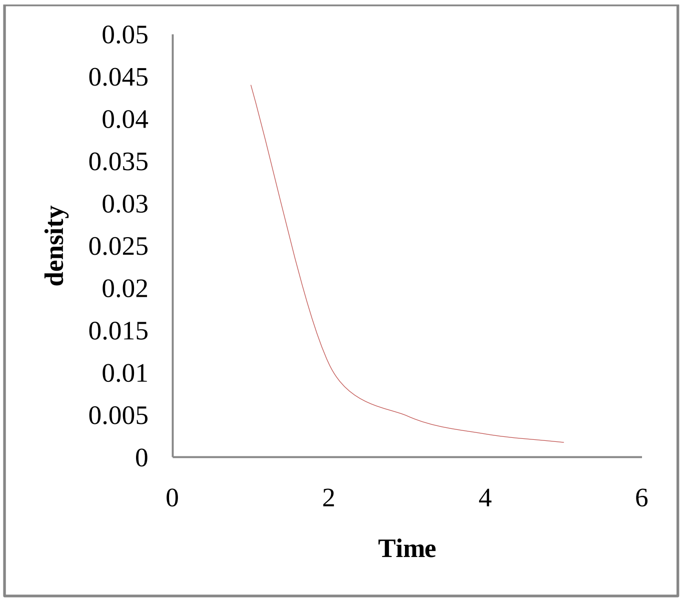

Fig 5: The plot of density (ρ) verses time (t), α=1, β=5.

From the figure,it is observed that at an initial epoch Proper energy density diverges. As cosmic time t increases gradually it decreases and finally vanishes when t → ∞

String tension density,

$$\lambda = \frac{9[(\alpha-3)-\beta(\alpha-1)]}{4\alpha^2\beta(4\pi+\mu)t^2} \tag{4.6}$$

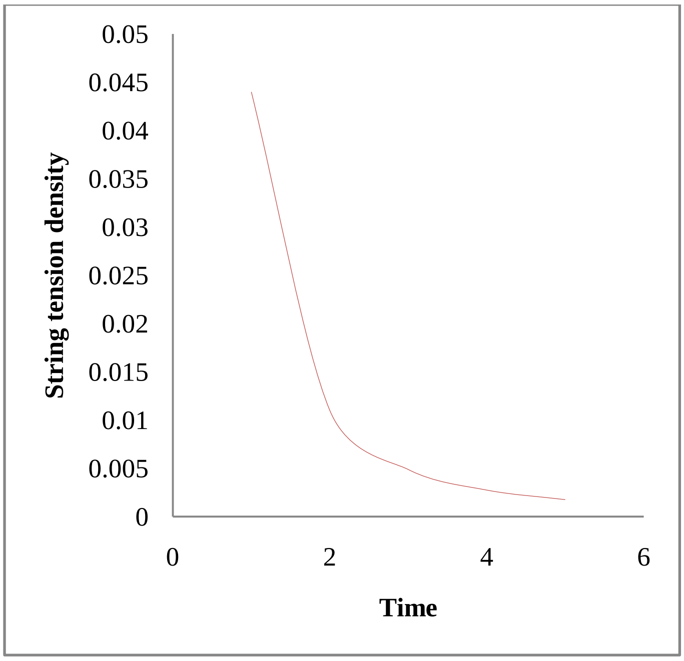

Fig 6: The plot of String tension density (λ) verses time (t), α=1, β=5.

From the figure,it is observed that at an initial epoch String tension density diverges. As cosmic time t increases gradually it decreases and finally vanishes when t → ∞.Hence it can be noted that, string tension density vanishes at present epoch that is why string disappears from present universe

Particle energy density,

$$\rho_p = 0 \tag{4.7}$$

Particle energy density gets vanished in this case, i.e.in case of geometric (Nambu)string.

Ratio of anisotropy,

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} \neq 0 \tag{4.8}$$

Mean anisotropic parameter is,

$$A_m = \frac{3(\beta^2-2\beta+3)-2\alpha\beta^2(\alpha t^2+2)}{2\alpha^2\beta^2 t^2} \tag{4.9}$$

Ratio of anisotropy, $\frac{\sigma^2}{\theta^2} \neq 0$ and Mean anisotropic parameter, $A_m \neq 0$ shows that

Model does not attain isotropy i.e. it is anisotropic throughout the whole evolution of the earth.

Case (ii) Reddy String

Reddy use equation of state for string density as,

$$\rho + \lambda = 0 \tag{4.10}$$

i.e. sum of the rest energy density and tension density for a cloud of a string vanishes .This relation is analogues to the relation $\rho + p = 0$ i.e. equation of state in General Relativitywhich represent false vacuum .

In this case, we get same string model given by equation (3.9) with same physical properties excepting the following.

Isotropic pressure,

$$p = \frac{3(\beta-1)[(3(\beta-1))-2\alpha\beta-6]}{4\alpha^2\beta^2(4\pi+\mu)t^2} \tag{4.11}$$

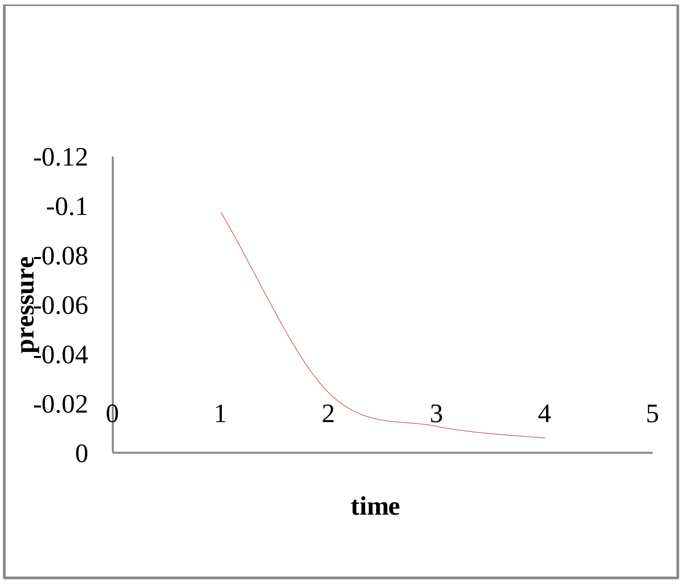

Fig 7: The plot of pressure (p) verses time (t), α=1, β=5.

Proper energy density,

$$\rho = \frac{3[6\beta(3-\beta)-\alpha\beta(3-2\beta)-9}{8\alpha^2\beta^2(4\pi+\mu)t^2} \tag{4.12}$$

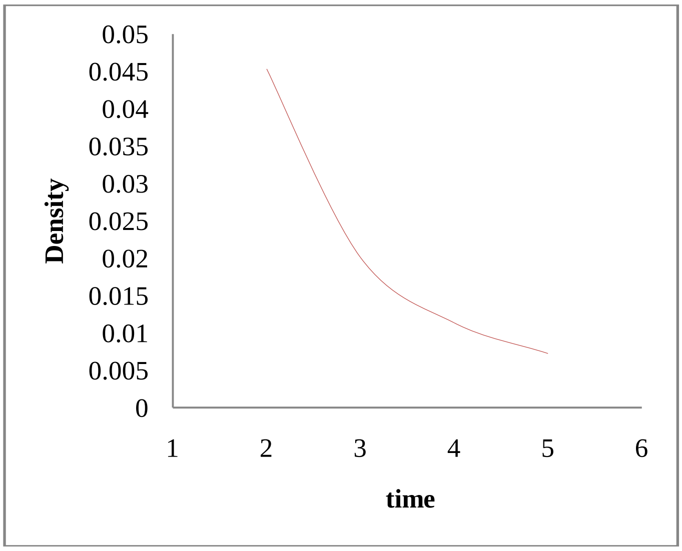

Fig 8: The plot of density (ρ) verses time (t), α=1, β=5.

String tension density

$$\lambda = -\frac{3[6\beta(\beta-3)-\alpha\beta(3-2\beta)-9}{4\alpha^2\beta^2(4\pi+\mu)t^2} \tag{4.13}$$

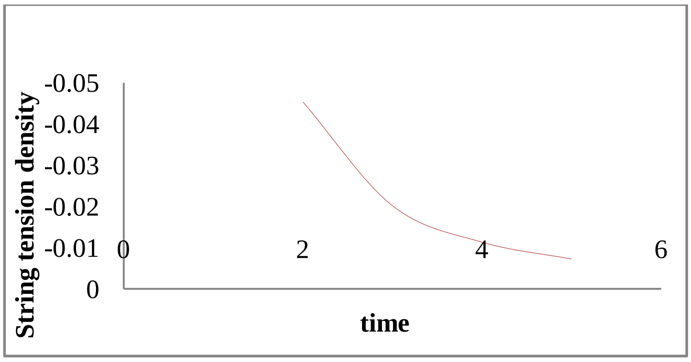

Fig 9: The plot of String tension density (λ) verses time (t), α=1, β=5.

Particle energy density,

$$\rho_p = \frac{3[6\beta(\beta-3)-\alpha\beta(3-2\beta)-9}{4\alpha^2\beta^2(4\pi+\mu)t^2} \tag{4.14}$$

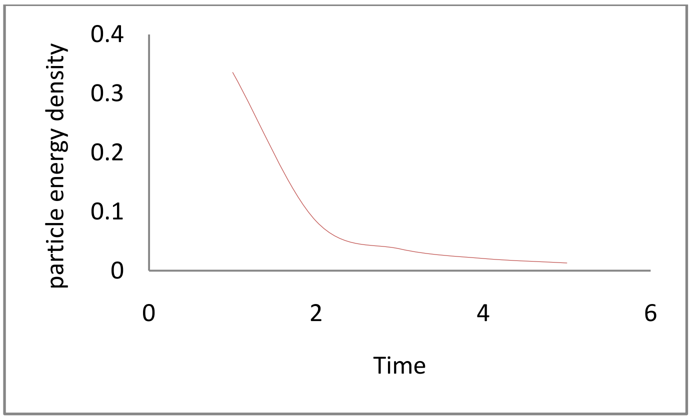

Fig 10: The plot of Particle energy density (ρp) verses time (t), α=1, β=5.

Since in this case we get same cosmological model and physical properties, physical behavior will also be similar as that of case of geometric (Nambu) string. Only, in this case, isotropic pressure, proper energy density and string tension density has different values and particle energy density does not get vanished and it has particular value given by (4.14).

CONCLUSION

We obtained axially symmetric anisotropic massive string cosmological models in f(R,T) theory of gravity .The model we obtained represent Nambu string and Reddy string in f(R,T) theory of gravity .It is important to note that models are identical in both cases. In our models, string density vanishes at present epoch that is why string disappears from present universe. At an initial value all physical parameter get diverges while for large value of t they all get vanishes.

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